

# PROOF OF GOLDBACH'S CONJECTURE.

NICETO VALCÁRCEL YESTE, BSc IN PHYSICAL SCIENCES FROM THE SPANISH  
NATIONAL DISTANCE EDUCATION UNIVERSITY (UNED).

July 2, 2018

Goldbach's Conjecture states that all even numbers greater than 2 can be expressed as the sum of two prime numbers.

Suppose that the Conjecture is not true and assume that there is at least one first even number  $2n$  ( $n \in \mathbb{N}$ ) that cannot be expressed as the sum of two prime numbers. How does this condition affect the preceding even number,  $2n - 2$ , for which the Conjecture must hold?

Let  $\{C_{2n}\}$  be the set of composite numbers  $C_j, C_i, C_k, C_h, \dots$  that are all less than  $2n$ , and let  $\{P_{2n}\}$  be the set of prime numbers  $P_j, P_i, P_{i'}, \dots$  that are all less than  $2n$ .

For  $2n$ , the following must be true:

- A)  $2n = C_h + C_k$
- B)  $2n = C_j + P_j$ , but not C)
- C)  $2n = P_i + P_j$ .

Subtracting 2 from both sides of the two equalities above gives:

- A')  $2n - 2 = C_h + C_k - 2$ .
- B')  $2n - 2 = C_j + P_j - 2$ .

The prime number  $P_j$  represents both a specific prime number and each and every prime number  $\in \{P_{2n}\}$  in general, such that if  $(C_j - 2)$  is a composite number  $\in \{C_{2n}\}$ , all of these prime numbers are symmetrical about  $\frac{2n-2}{2} = n - 1$  to a composite number, and thus the Conjecture would not hold.

If the number  $(C_j - 2)$  were a prime number  $P_j$ , this would also be the case for all prime numbers symmetrical to  $P_j$  about the number  $(n - 1)$ , which would mean:  $2n - 2 = P_j + P_j'$ , which is impossible, because this would require the prime numbers  $P_j$  to be distributed symmetrically to  $P_j'$  about the number  $n - 1$ , and that contradicts the results of the Prime Number Theorem ( $\Pi(x) \sim \frac{x}{\ln x}$ ) regarding the distribution of prime numbers on the real line.

Therefore, the Conjecture does not hold for the value  $2n - 2$ .

Because the initial assumption leads to a contradiction, it follows that the Conjecture is true.

I would like to thank the readers for their time and for their comments on this work, which can be sent to me at [nicetovalcarcel@gmail.com](mailto:nicetovalcarcel@gmail.com)