DEMONSTRATION OF THE TWIN PRIME CONJECTURE.

Niceto Valcárcel Yeste. BSc in Physical Sciences from the Spanish National Distance Education University (U.N.E.D.).

January 7, 2014

1 Introduction.

A pair of twin primes p_1, p_2 consists of two prime numbers such that:

$$p_2 = p_1 \pm 2$$

i.e. they have a difference of two.

The Conjecture is so called because it is not known whether there is an infinite number of these twin primes.

The aim of this paper is to provide a solution for this conjecture.

IN is the set of natural numbers including 0.

*IN** is the set of natural numbers excluding 0.

IP is the set of prime numbers.

An essential instrument used in this paper is the set of non-prime or composite odd numbers, completely identified, with no particular features of any class, based on a set defined here as $\{m\}$ of the natural numbers m from which the non-prime odd numbers are obtained:

$$\{m\} = \{m \in IN \,/\, (2m + 1) \notin IP\}$$

This demonstration begins to be developed in section 2, defining the set $\{m\}$ and the set $\{m + 1\}$, which is constructed by adding 1 to the elements of $\{m\}$.

This is continued in section 3, which proposes and demonstrates a biconditional, i.e. a condition of the type "if and only if" ($\Leftrightarrow$), between the truth of the Conjecture and the set obtained from the union $\{\{m\} \cup \{m + 1\}$. The condition in question is:

The Conjecture is false "if and only if" there is a natural number n_0 such that, for every $n \in IN$, $(n_0 + n) \in \{\{m\} \cup \{m+1\}\}$.

A set that meets this condition could be denoted as a "*continuous set of natural numbers*" (hereinafter, *csnn*), in the sense

that it contains a number such that all greater natural numbers also belong to this set.

Thus, any finite set of numbers can be removed from a *csnn* in such a way that this set will continue to be a *csnn*, and at the same time, the *csnn* will continue to be a *csnn* if it is translated by n_i units ($n_i \in IN*$). Translating a set by n_i units means adding the value n_i to each of its elements.

This paper ends by demonstrating the truth of the Conjecture by reductio ad absurdum.

It is assumed that the Conjecture is false and, after unit translation of the set $\{\{m\} \cup \{m + 1\}\}$ by adding 1 to each of its elements, a false result is obtained. This contradicts the relevant results of the general Prime Number Theorem.

2 The set {*m*}.

Any non-prime odd number $(2m + 1)$ can be expressed in general as:

$2m + 1 = (2x + 1)(2y + 1) = 2(2xy + x + y) + 1$

for every pair of values $(x, y) \in IN*$.

If $2m + 1$ is a composite number, it is the product of at least two numbers other than 1. If it is the product of more than two numbers, based on the associative property of multiplication, it can always be converted into a product of two numbers other than 1.

Furthermore, the product of two numbers under this condition is always a composite number.

The set $\{m\} = \{2xy + x + y\}$ is the set of natural numbers m, from which all non-prime odd numbers are obtained.

The set $\{m + 1\}$ is the set obtained from $\{m\}$ by adding 1 to each of its elements:

$\{m + 1\} = \{2xy + x + y + 1\}$.

The subset $\{m + 1\}_p \subset \{m + 1\}$ is the subset of elements of $\{m + 1\}$ that are not elements of $\{m\}$, and therefore, for those, $(2(m + 1)_p + 1)$ is a prime number.

3 Proposition.

The Twin Prime Conjecture is false if and only if there is a number $n_0 \in IN$ such that for every $n \in IN$, the number $(n_0 + n) \in \{\{m\} \cup \{m + 1\}\}$.

3.1 Demonstration.

$\Rightarrow$)

If the Conjecture is false, then there is a last pair of twin primes.

Let this be the pair consisting of $(2n_0 - 3)$ and $(2n_0 - 1)$.

- If the number: $(2(n_0 + n) + 1)$ is not prime $\Rightarrow (n_0 + n) \in \{m\}$.

- If the number: $(2(n_0 + n) + 1)$ is prime $\Rightarrow (n_0 + n) \notin \{m\}$,

and the preceding and following odd numbers are not prime:

$(2(n_0 + n - 1) + 1)$ and $(2(n_0 + n + 1) + 1)$

and therefore,

$(n_0 + n - 1) \in \{m\} \Rightarrow (n0 + n) \in \{m + 1\}$.

$\Leftarrow$)

If there is a number n_0 such that for every $n \in IN$, the number $(n_0 + n) \in \{\{m\} \cup \{m + 1\}\}$, then the Conjecture is false.

- If $(n_0 + n) \in \{m\}$, $(2(n_0 + n) + 1)$ is not prime and will not be part of a pair of twin primes.

- If $(n_0 + n) \in \{m + 1\}_p$, $(2(n_0 + n) + 1)$ is prime, and the following statements are true:

a) $(n_0 + n) \notin \{m\} \Rightarrow (n_0 + n) \in \{m + 1\} \Rightarrow (n_0 + n - 1) \in \{m\}$.

The number $(2(n_0 + n - 1) + 1) = 2(n_0 + n) - 1$ is not prime.

b) $(n_0 + n) \notin \{m\} \Rightarrow (n_0 + n + 1) \notin \{m + 1\} \Rightarrow (n_0 + n + 1) \in \{m\}$.

The number $(2(n_0 + n + 1) + 1) = 2(n_0 + n) + 3$ is not prime.

Conclusion: $(2(n_0 + n) + 1)$ is not one of a pair of twin primes.

4 Proposition.

The Twin Prime Conjecture is true.

4.1 Demonstration by reductio ad absurdum.

Suppose the Conjecture is false, and therefore:

$$\exists\, n_0 \in IN \,/\, \surd\, n \in IN,\ (n_0 + n) \in \{\{m\} \cup \{m + 1\}\}$$

where $(2n_0 - 3)$; $(2\, n_0 - 1)$ is the last pair of twin primes, which implies that: $\{(n_0 - 2); (n_0 - 1)\} \notin \{m\}$.

After unit translation of the elements:

$(n_0 + n) \in \{\{m\} \cup \{m + 1\}\}$; $(\surd\, n \in IN)$

it must hold that:

$\exists\, (n_0 + 1) \,/\, \surd\, n \in IN,\ (n_0 + 1 + n) \in \{\{m + 1\} \cup \{m + 2\}\}$

implying that the unit translation consists of adding 2 to the elements $(n_0 + n) \in \{m\}$; $(n \in IN)$.

- After the unit translation, then:

$[(n_0 + 1 + k) \in \{m + 1\}_p\,;\ (k \in IN)]$

is any number greater than or equal to $(n_0 + 1)$.

Therefore, $[2(n_0 + 1 + k) + 1]$ is any prime number greater than or equal to $(2n_0 + 3)$.

As a result: $[(n_0 + 1 + k \pm 1) \in \{m\}]$, because there are no twin primes greater than $(2n_0 - 1)$.

It therefore holds that: $[(n_0 + 1 + k + 2) \in \{m + 1\}_{\mathrm{p}}]$, because there are only two ways in which $(n_0 + 1 + k + 2) \in \{\{m + 1\} \cup \{m + 2\}\}$ can be true:

1) if $(n_0 + 1 + k) \in \{m\}$, because the translation transforms this into $(n_0 + 1 + k + 2)$. This proposition is impossible because $(2(n0 + 1 + k) + 1)$ is prime.

2) if $(n_0 + 1 + k + 2) \in \{m + 1\}_p$, because the elements of $\{m + 1\}_p$ are not changed by the translation.

Therefore, it must hold that: $[(n_0 + 1 + k + 2) \in \{m + 1\}_p]$ and $[(n_0 + 1 + k \pm 1) \in \{m\}]$

The condition obtained above, where $(n_0 + 1 + k + 2)$ must be a member of the set $\{m + 1\}_p$, requires this same membership condition for all $[(n_0 + 1 + k + 2k'); (k' \in IN*)]$, as shown below.

$(n_0 + 1 + k + 2) \in \{m + 1\}_p \Rightarrow (n_0 + 1 + k + 3) \in \{m\}$, as there are no pairs of twin primes greater than $(2n_0 - 1)$.

Furthermore, if $(n_0 + 1 + k + 3) \in \{m\} \Rightarrow (n_0 + 1 + k + 4) \in \{m + 1\}_p$, because there are only two ways in which $(n_0 + 1 + k + 4) \in \{\{m + 1\} \cup \{m + 2\}\}$ can be true:

1) if $(n_0 + 1 + k + 2) \in \{m\}$, because the translation transforms this into $(n_0 + 1 + k + 4)$. This proposition is impossible because $(2(n_0 + 1 + k + 2) + 1)$ is prime.

2) if $(n_0 + 1 + k + 4) \in \{m + 1\}_p$, because the elements of $\{m + 1\}_{\mathrm{p}}$ are not changed by the translation.

Thus, it must hold that: $[(n_0 + 1 + k + 4) \in \{m + 1\}_p]$ and $[(n_0 + 1 + k + 4 \pm 1) \in \{m\}]$.

And so on indefinitely.

The conclusion obtained is false, contradicting the results of the general Prime Number Theorem regarding the distribution of prime numbers; specifically, when $X \rightarrow \infty$:

$$\prod(x) = \frac{x}{\ln x}$$

It also contradicts the distribution of multiples of 3, 5, 7, etc. on the real line.

Because the proposition that was assumed to be true leads to a false conclusion, the proposition must be false, and therefore the Conjecture is true.

I would like to thank the reader for their time and for their comments on this work, which can be sent to me at:

nicetovalcarcel@gmail.com