IRRATIONALITY OF THE NUMBERS: π/*e* AND π*e*.

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INTRODUCTION.

The purpose of this paper is to demonstrate the irrationality of the numbers $\frac{\pi}{e}$ and πe, first proving that at least one of them is irrational and then, based on this proof, showing that the other is too.

PROOF.

Suppose that both numbers are rational and, therefore, that there are integers (*a, b, c, d*) such that:

$a)\ \frac{\pi}{e} = \frac{a}{b}$.

$b)\ \pi \mathrm{e} = \frac{c}{d}$.

Dividing equation *b*) by equation *a*) gives:

$e^2 = \frac{cb}{ad}$.

Taking the Napierian logarithm of both sides of the previous equation gives:

$2 = L\frac{cb}{ad}$, which is impossible because the Napierian logarithm of a rational number is an irrational number.

Therefore, at least one of the two $\left(\left(\frac{\pi}{e}\right) \text{ or } (\pi e)\right)$ is an irrational number *i*.

Let:

$a_1)\ \frac{\pi}{e} = \frac{a}{b}$.

$b_1)\ \pi e = i$.

Let γ(*k*) be the decimal digits of a number *k* starting from any decimal place, as near to or as far from the integer part of *k* as desired.

From equation a_1), we find:

$\gamma\left(b\frac{\pi}{e}\right) = \gamma(a) = 0$.

From equation b_1), we find:

$\gamma(\pi e) = \gamma(\mathrm{i})$.

Multiplying these two expressions gives:

$\gamma\left(b\frac{\pi}{e}\right)\gamma(\pi e) = 0 = \gamma_1\left(b\frac{\pi}{e}\pi e\right) = \gamma_1(b\pi^2)$, which is not equal to zero.

This therefore proves that both numbers $\left(\left(\frac{\pi}{e}\right) \text{ and } (\pi e)\right)$ are irrational.

I would like to thank the readers for their time and their comments on this work, which can be sent to me at: nicetovalcarcel@gmail.com