DEMONSTRATION OF THE FERMAT PRIME CONJECTURE.

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INTRODUCTION.

The aim of this paper is to demonstrate whether there is an infinite number of Fermat primes.

A Fermat number is a number in the form $2^{2^n} + 1$, where n is a natural number greater than 1.

This proof will suppose that there is a last Fermat prime number, and this will lead to a false conclusion.

PROOF.

Let $2^{2^n} + 1$ be the last Fermat prime number.

Therefore, there will be no pair of natural numbers (x, y) such that:

$2^{2^n} + 1 = (2x + 1)(2y - 1)$

This condition can be expressed in another way. Operating on the preceding equation:

$2^{2^n-1} + 1 = 2xy - x + y$

If this is the last Fermat prime number, then there must be a pair of natural numbers (x_j, y_j) such that for any k ∈ ℕ*, the following equation holds:

$2^{2^n+k} + 1 = (2x_1 + 1)(2y_1 - 1) = 4x_1y_1 - 2x_1 + 2y_1 - 1$

Operating on this equation:

$2^{2^n+k-1} + 1 = 2x_1y_1 - x + y_1 = (2x_2 + 1)(2y_2 - 1)$

Operating on this last equation:

$2^{2^n+k-2} + 1 = 2x_2y_2 - x_2 + y_2$

By performing this operation $k + 1$ times, we obtain:

$2^{2^n-1} + 1 = 2x_{k+1}y_{k+1} - x_{k+1} + y_{k+1}$, which is false.

As the proposition that was assumed to be true leads to a false conclusion, we conclude that the number of Fermat prime numbers is infinite.

I would like to thank the reader for their time and for their comments on this work, which can be sent to me at:

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