

DEMONSTRATION OF THE MERSENNE PRIME CONJECTURE.

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INTRODUCTION.

The aim of this paper is to demonstrate whether there is an infinite number of Mersenne primes.

A Mersenne number is a number in the form $2^n - 1$, where n is a natural number greater than 1.

This proof will suppose that there is a last Mersenne prime number, and this will lead to a false conclusion.

PROOF.

Let $2^n - 1$ be the last Mersenne prime number.

Therefore, there will be no pair of natural numbers (x, y) such that:

$$2^n - 1 = (2x + 1)(2y + 1)$$

This condition can be expressed in another way. Operating on the preceding equation:

$$2^{n-1} - 1 = 2xy + x + y$$

If this is the last Mersenne prime number, then there must be a pair of natural numbers (x_j, y_j) such that for any $k \in \mathbb{N}^*$, the following equation holds:

$$2^{n+k} - 1 = (2x_1 + 1)(2y_1 + 1) = 4x_1y_1 + 2x_1 + 2y_1 + 1$$

Operating on this equation:

$$2^{n+k-1} - 1 = 2x_1y_1 + x_1 + y_1 = (2x_2 + 1)(2y_2 + 1)$$

Operating on this last equation:

$$2^{n+k-2} - 1 = 2x_2y_2 + x_2 + y_2$$

By performing this operation $k + 1$ times, we obtain:

$$2^{n-1} - 1 = 2x_{k+1}y_{k+1} + x_{k+1} + y_{k+1}, \text{ which is false.}$$

As the proposition that was assumed to be true leads to a false conclusion, we conclude that the number of Mersenne prime numbers is infinite.

I would like to thank the reader for their time and for their comments on this work, which can be sent to me at:

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