PROOF TO ONE OF LANDAU'S FOUR PROBLEMS.

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1. INTRODUCTION.

This paper tries to demonstrate the infinity of numbers of the form $\left(n^2+1\right)$, and for this, we will obtain another larger prime number and in the same way$(n+k)^2+1$ where $k=\Pi p_j$ is the product of every prime numbers $<\left(n^2+1\right)$.

2. DEMONSTRATION.

Let $\left(n^2+1\right)$ the last prime number of this way
$(n+k)^2+1=\left(n^2+1\right)+k\,(k+2n)$
Dividing by any $p_j \in \Pi p_j \Longrightarrow \frac{\left(n^2+1\right)+k(k+2n)}{p_j}=\frac{n^2+1}{p_j}+\frac{k(k+2n)}{p_j}$
$\frac{k(k+2n)}{p_j}$ is a natural number beaucause k contains to any $p_j \in \Pi p_j$, but $\frac{n^2+1}{p_j}$is a rational number.
Therefore $(n+k)^2+1$ is a prime number $>\left(n^2+1\right)$

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